

# Correcting Circular Error

Robert Matthys proposes a novel solution to a common problem

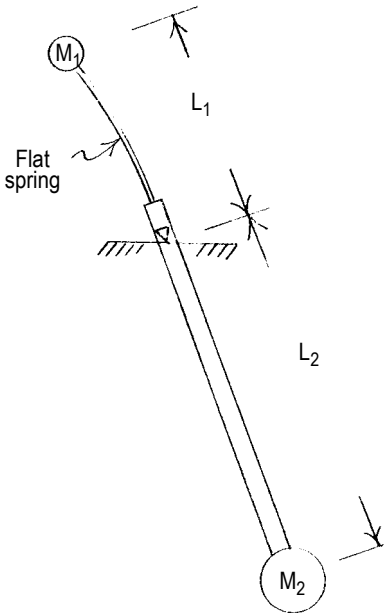
IN THIS ARTICLE a way to correct a pendulum's period (circular) error for semi-amplitudes up to at least 3° is described.

The concept was first suggested by Woodward<sup>1</sup> as a variant on an earlier idea by White<sup>2</sup>. The idea is to mount a second bob at the end of a long flat spring, which is in turn mounted as an upside down pendulum on the top end of a simple pendulum, 1. When the pendulum swings, the upper spring bends and allows the upper bob to deflect farther from the vertical than the lower bob, and thereby corrects for period error. Traditionally, the variation in period with the amplitude of swing has been called 'circular error', but I prefer the more accurate term 'period error'.

## Compound Pendulums

With the correction spring and a second bob, the simple pendulum becomes a compound one. A compound pendulum is defined here as having part of its mass above the pendulum's axis of rotation, whereas a simple pendulum has all of its mass below the axis of rotation. The advantage of a compound pendulum is that you can get long periods with a short pendulum length. That advantage won't occur here, as the aim is to cancel out the period error rather than get a longer period or a shorter pendulum. A disadvantage is that the compound pendulum's Q is lower than that of a simple pendulum<sup>3</sup>.

Compound pendulums are more complicated than simple ones. In a compound pendulum, the most important design factors are the moments of torque<sup>3</sup> of the upper and lower bobs about the pendulum's axis of rotation. Bob weights and pendulum lengths are not important in themselves, but only in how they affect the moments of torque, i.e., the product of a bob's weight and its distance from the pendulum's axis of rotation. The inertia of the bobs about the axis of rotation may also be important. The two equations<sup>4, 5</sup>



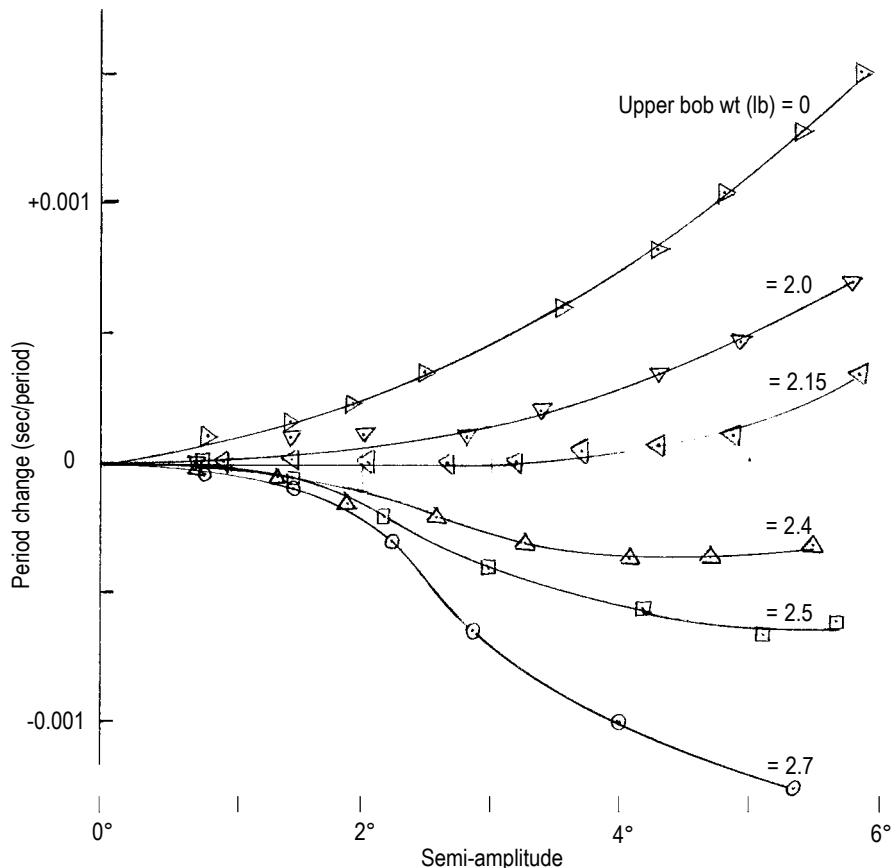
1. Period error correction concept: an inverted pendulum of mass  $M_1$  mounted on the top of long flat spring.

I found for a compound pendulum's period were different. One source gave only the final equation, and the other gave the final equation along with part of its derivation. The period equation for a compound pendulum needs to be derived, or at least verified, with more rigour by a mathematician.

## Experiments

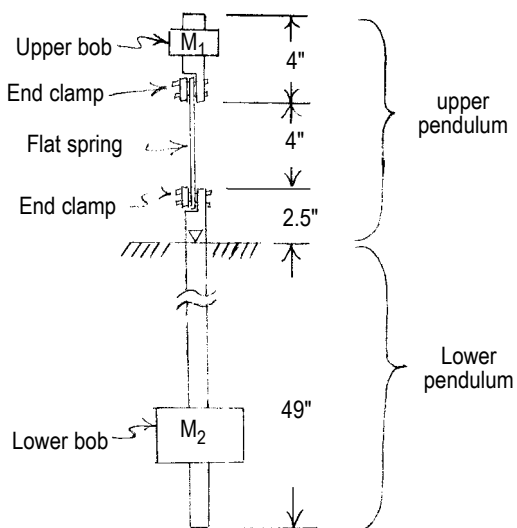
One true rule is that the torque moment of the upper bob cannot exceed that of the lower bob, or the pendulum will turn over, end for end. Experimentally, I have found that moving either the upper or lower bob farther away from the axis of rotation will increase the period. And any weight added above the axis of rotation also will increase the period. In contrast to the simple pendulum, the weight of the compound pendulum's lower bob does affect the period. Increasing the lower bob's weight will decrease the period.

A useful shorthand notation for the compound pendulum is: 'upper bob, 2.2lb at 8.7"', which means a bob weight of 2.2 pounds (1kg) located 8.7 inches above the



2. Period change vs semi-amplitude for a compound pendulum with a spring-mounted upper bob. Period is 2s at zero amplitude. Upper spring 4x4x0.032" (LWT), rod 3/8" steel threaded, upper bob (variable) at 8.7", lower bob 19lb at 36.5-38.5".

- Woodward, P. 'Compensation of Pendulums', *Horological Journal*, **125** (7) [January 1986], p.10 (note: pendulum drawing is printed upside down).
- White, I. 'Compensation of Pendulums', *Horological J.*, **125** (5) [Novem. 1985]p. 24.
- Matthys R. 'Real Compound Pendulums', *NAWCC Horological Science Newsletter*, September/October, 2004.
- Miller, D. 'Playing around with Pendulums', *Clockwise* pp.11-12 April, 26-27 May, 1980.
- Hagans, O. *Best of J E Coleman: Clockmaker*, AWI, 2003.



3. Test pendulum.

axis of rotation. Or, 'lower bob, 19lb at 30"', which means a lower bob weight of 19 pounds at 30 inches below the axis of rotation. All of the swing angles in this article are half angles, i.e., semi-amplitudes.

Figure 2 shows the period error for a compound pendulum that uses a flat spring 4" long (10cm) as the upper pendulum rod. The lower bob is 19lb at 36.5-38.5" and the upper bob is 0-2.7lb at 8.7". With an upper bob weight of 2.15lb the period error is compensated to zero for semi-amplitudes up to 3°.

The period was measured electronically<sup>6</sup> using interchangeable 0.5 to 3" wide black metal 'flags' attached to the bottom of the pendulum rod. The flags interrupt a thin (0.02") light beam located at the center of swing. At 6° semi-amplitude the flag used is six times wider than at 1° semi-amplitude, so as to get approximately the same flag width in units of time.

The relevant mechanical details of the pendulum are shown, 3. The actual pendulum suspension used was two flat springs each 1/4 x 3/8 x 0.006" (LWT) in parallel, rather than the knife edge shown for simplicity in 3. There is a short piece of rod above the upper spring to allow small adjustments to the period correction. For each curve in 2 the weight of the upper bob is changed, but not its location. The lower pendulum was left alone, except to move the lower bob slightly to maintain the zero amplitude period at two seconds. Adding or subtracting weight to or from the upper bob changes the period, which is brought back to two seconds each time by moving the lower bob up or down a little as needed.

6. Matthys, R. 'Measuring a Pendulum's Period', *NAWCC Horological Science Newsletter*, July 2004 pp.3-15.

The upper flat spring used in 2 was brass, 4x4x0.032" (LWT). I had brass sheets 1/16 and 1/32" thick on hand to make springs. The 4" width looks awkward, but works well as a test spring. On a real clock, one would use a narrower and thicker spring of equal stiffness.

For a given bob weight, the spring deflection is proportional to the length (cubed,) divided by the product of the width, the thickness (cubed), and the modulus of elasticity.

Assuming equal moduli, equal deflection would be given by narrower and thicker springs of size 4x2x0.040", 4x1x0.050", or 4x0.5x0.063" (LWT).

Since the material was on hand, the narrowest and thickest of the three sizes was tried and was satisfactory on the pendulum, but it had poor handling qualities because of the higher stress level when being handled. About half the time, the 0.063" spring would bend too far and take a permanent set when the pendulum was picked up from a horizontal position. The 4" wide brass spring handled well and didn't do that, but it was thinner (0.031").

A longer spring or one of intermediate thickness is called for. For springs of equal deflection, the internal stress level increases linearly with thickness. Table 1 lists the maximum internal bending stress for several springs of equal deflection, under the condition of the pendulum laying horizontal and slowly being lifted off the bench.

Table 1. Max bending stress in the long upper spring when the pendulum is in a horizontal position. All four springs are type 510 Phosphor Bronze and provide equal deflection.

Size (LxWxT)".	Maximum stress (psi)
4x4x0.032	12,400
4x2x0.040	15,900
4x1x0.050	20,400
4x0.5x0.063	26,100

The most satisfactory spring materials for this application are type 172 Beryllium Copper (best) and type 510 Phosphor Bronze (second best). They are available in many thicknesses. In particular, 510 Phosphor Bronze is available in flat sheets 0.040" and 0.050" thick.

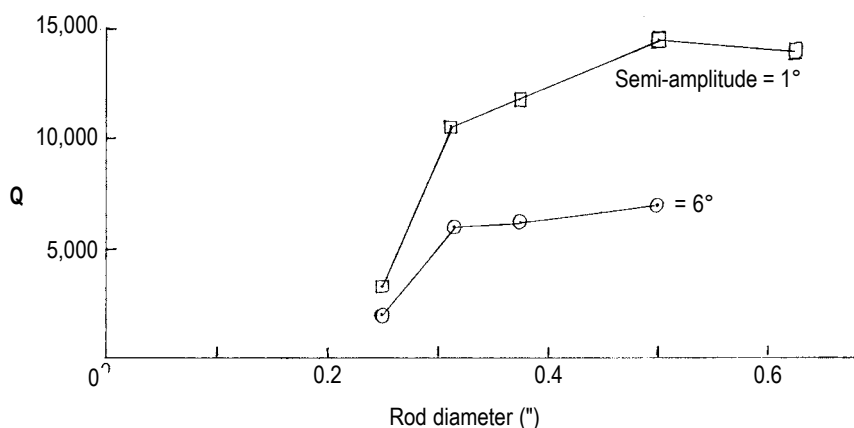
The upper bob and the long upper spring form a spring-mass system with its own resonant frequency. For stable operation of a pendulum with a two second period, this resonant frequency should be 1.5-3Hz. The higher it is, the more stable the pendulum is. If this resonance is down about 1.0Hz, the upper pendulum bounces around continuously and erratically, and never settles down. At 1.5-3Hz, the upper pendulum settles down in a minute or so, and flops gently back and forth in time with the motion of the lower pendulum.

The lower pendulum rod needs to be extra stiff. This is due to the cyclic rod bending torque introduced by the upper bob<sup>3</sup>. The cyclic rod bending is an energy loss, and lowers the Q. Figure 4 shows how the Q varies with rod diameter, where all the rods are steel with an external thread. The maximum Q occurs at the 0.5" diameter. Rod surface area and air drag increase as the diameter increases. At 5/8" diameter, the air drag apparently is big enough to lower the Q a little, making the 1/2" diameter the point of maximum Q.

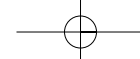
Conclusions

In summary, 2 shows that the period error can be reduced to zero for swing amplitudes up to at least 3°. The reduction is obtained by adding an upside down pendulum to the top of a simple pendulum. The upside down pendulum consists of a bob mounted at the top end of a long flat spring. For a lower bob weight of 19lb the optimum upper bob weight is 2.15lb, about 11% of the weight of the lower bob, 2.

How well this idea will work over time depends on how stable the long upper



4. Q vs pendulum rod diameter and semi-amplitude. Period is 2s, upper spring 4x4x0.031" (LWT), upper bob 2.15lb at 9.4", lower bob 19lb at 37 to 38".



spring is over time. At 3° semi-amplitude, the spring is correcting for a period error of about 450 microseconds out of 2 seconds, or about 2.2 parts in 10<sup>4</sup>. If the spring were to lose stiffness by (say) 1%, that would be nominally equivalent to a 1% increase in upper bob mass. Interpolation between the curves in 2 shows that that would decrease the two second period by about 25 microseconds, or 2.3 seconds per day. At a smaller semi-amplitude of 1°, 2, shows that the error would be about six times smaller, or 0.4 seconds per day. The spring is obviously going to have to be either very stable or swing at a low amplitude if the time error caused by any instability in the spring is to be kept small.

In the Fedchenko pendulum (not described here), the period error is also compensated to zero, using a different compensation approach involving its suspension springs. The Fedchenko pendulum also has a long-term stability problem (in its suspension springs), similar to the one described here.

□

**Additional reading:**

7. Fenner, R. 'A Seconds Pendulum Using Circular Error Compensation', *HJ*, 139 (8) [August 1997] pp.262-265.
8. Pipes, M 'Reducing Circular Error', *HJ*, 141 (9) [September 1999] pp.307-310.
9. Clifford, C. 'Circular Error Compensation', *Horological Journal* 91 (1092) [September 1949] pp.542-543.

